Electron kinetics in isolated mesoscopic rings driven out of equilibrium

V.I.Yudson[1,2]
[1] *Center for Frontier Science, Chiba University, 1-33 Yayoi-cho, Inage-ku, Chiba 263-8522, Japan*
[2] *Institute of Spectroscopy, Russian Academy of Sciences, 142090, Troitsk, Moscow region, Russia*

V.E Kravtsov[3,4]
[3] *International Centre for Theoretical Physics, P.O. Box 586, 34100 Trieste, Italy*
[4] *Landau Institute for Theoretical Physics, Russian Academy of Sciences, 2 Kosygina str., 117940, Moscow, Russia*

(Dated: February 1, 2008)

Kinetic theory of nonlinear current response to an external field is developed for mesoscopic normal metal rings threaded by a magnetic flux. General expressions for direct current (DC) are derived for a non-equilibrium regime. These expressions describe simultaneously a contribution to DC made by a non-equilibrium (external) field, as well as the contribution caused by interaction with intrinsic fields. Contributions of electron-electron and electron-phonon interactions to the direct current in a non-equilibrium systems are estimated. The kinetic equation for electrons in a pumped disordered metal ring is solved with taking into account the Coulomb and electron-phonon interactions. This gives an estimate of an overheating of the pumped electron system.

PACS numbers: 72.15.Rn, 72.40.+w, 72.70.+m, 72.20.Ht, 73.23.Ra, 73.23.-b

I. INTRODUCTION

Usual objects for experimental and theoretical studies of mesoscopic phenomena in linear or non-linear conductivity are systems with two or more external leads. Because of experimental difficulties, mesoscopic systems without leads have received less attention, except for a long-standing puzzle of persistent currents (PC) in normal metal rings in the Aharonov-Bohm configuration[1,2,3,4,5,6,7]. As the persistent current is an equilibrium mesoscopic phenomenon, most of theoretical papers deal with the thermodynamical properties of the electron system; it is assumed usually that the thermodynamic equilibrium has been sustained in the experimental investigations cited.

Although an early theoretical study[8] of non-equilibrium conductivity in mesoscopic rings has predicted a peculiar effect of a direct current (DC), which is induced by an external alternating (ac) electromagnetic field, there has not been performed experimental search for this effect until very recently[6].

The interest to non-equilibrium mesoscopic phenomena in systems without leads has increased recently in connection with a hypothesis[9,10] that the observed magnitudes of PC may include a large DC contribution, induced by an uncontrolled external noise field. It has been supposed also[9,11] that this uncontrolled noise may be the reason of the anomalous electron dephasing rate discovered in experimental studies[4] of weak localization corrections to the conductance of long wires. Comparing the dephasing rate caused by an external electromagnetic field[12] in long wires, and the magnitude of DC current[8] induced in a ring by the external field with the same parameters (both quantities are linear in the external field intensity), a universal relationship between the two quantities has been proposed recently[10]. However, the situation in this field remains still unclear and requires further research, both experimental and theoretical.

This set of problems together with the arising experimental interest, necessitate a deeper study of quantum kinetical processes in systems without external leads. The present paper is devoted to this study. An important physical difference between systems with and without leads is that in systems with leads, one may consider electron transport through the system even neglecting processes of inelastic scattering of electrons. The role of a "thermostat" may be played by scattering processes of electrons within the leads or in the external part of the electric circuit. On the contrary, in systems without leads (like metal rings) the meaning of a steady-state response to an external field is not well defined unless dissipation processes are taken explicitly into account.

In an earlier paper[13] we have considered these features of small mesoscopic systems in the situation where the main channel of the energy dissipation is provided by the electron exchange with neighboring reservoirs, so that the system allows for a simplified "dynamical" description in terms of the free electron model. Here we develop a self-consistent kinetic theory of nonlinear current response to an external field in mesoscopic normal metal rings threaded by a magnetic flux. The theory takes into account inelastic electron scattering processes, first of all the processes of the electron-phonon interaction, which are responsible for establishing a steady-state regime in the excited electron system. With the use of the Keldysh technique we have derived a generic expression for the direct (zero-frequency) current, which includes terms of two kinds: "kinetic" terms vanishing in the equilibrium and "thermodynamic" terms, which describe the interaction induced correction to PC. These two groups of terms have different analytical structure, that corresponds to the physical difference between retarded nature of the

response current and equilibrium processes; the latter allow for a weighted combination of emission-absorption processes without causality requirement for "absorption to precede emission"[14].

This approach provides a regularized description of the system. Moreover, it allows to resolve problems connected with anomalous "diffusion" modes of zero frequency and momentum (in ref.[13] we call them "loose diffusons"): we show that for the distribution function obeying the steady-state kinetic equation, the contribution of "zero-diffusion" modes vanishes identically (see Appendix). However, this modes are important in transient regimes.

In the presence of an external field the difference between PC and DC parts of the zero-frequency current has a somewhat conventional meaning. This is due to the abovementioned strong non-linearity of the electron system, which restricts validity of the formal expansion into powers of the external field. In fact, the PC part of the zero-frequency current, defined formally as proportional to the zero-th power of the external field, is modified as compared to its equilibrium value. This occurs due to a field-induced change of the steady-state distribution function of electrons. Such an "incoherent" kinetic effect is an unavoidable consequence of the external field action.

A steady-state regime in systems without leads may only be achieved via a mechanism of energy transfer from the pumped electron system to an external thermostat. As is usually assumed, this transfer is provided by electron-phonon interaction. The efficiency of this mechanism decreases with decreasing temperature that may lead to an overheating of the electron system (note, that more efficient inelastic electron-electron collisions conserve energy of electrons and therefore cannot provide cooling of electron system).

Study of field-induced modifications of the state of electron system is the major task of the present paper. In Sec. II we consider in detail the kinetic equation (KE) for electrons excited by an external field and cooled via interaction with a phonon bath. We have found a solution to this KE in the limit of relatively frequent inelastic electron-electron collisions. In this adiabatic limit the state of electron system may be considered as a quasi-equilibrium one, described by the Fermi distribution function with an effective temperature. This state relaxes slowly to a steady-state regime via electron-phonon inelastic scattering processes. We have found an explicit expression for the steady state electron temperature. We analyze also to what extent the absence of a global equilibrium between electron and phonon systems may modify the value of the direct current.

The paper has the following structure. General consideration of equilibrium and non-equilibrium direct current is carried out in section II. Model description, notations and formulation of the problem are given in the subsection II A. Formal expressions for various contributions to the direct current are derived by means of the Keldysh technique in subsection II B.

In section III we study a field-induced non-equilibrium energy distribution of electrons. Kinetic equation for electrons with taking into account electron-electron and electron-phonon interaction is described in subsection III A. A steady state solution to this equation in the adiabatic regime and the electron temperature relaxation rate are given in the subsection III B. A field-induced overheating of electrons and its physical consequences are discussed in subsection III C.

In section IV we estimate additional contributions to direct current caused by the absence of a complete equilibrium between electrons and phonons (subsection IV A) and by a deviation of electron distribution function from the equilibrium one (subsection IV B). We show that the latter contribution vanishes in the constant density of states approximation (for systems with electron-hole symmetry) while the former one is quite small. These study justifies the applicability of a free electron model for description of DC induced by an external field of a moderate intensity.

In section V we summarize the obtained results.

II. EQUILIBRIUM AND NON-EQUILIBRIUM DIRECT CURRENT

A. A model and formulation of the problem

We consider a normal metal ring of circumference L and cross-section area S, threaded by a magnetic flux φ. The ring is assumed to be thin, i.e. $\sqrt{S} \ll L$. It is assumed also that $L \gg l$, where l is the electron mean free path with respect to elastic scattering by impurities. For a time independent magnetic flux φ, the ring is known to possess an equilibrium persistent current I_{PC}. A time-dependent magnetic flux would lead to the direct (zero-frequency) current along the metal ring, this current is a nonlinear response to the external perturbation[8].

Before specifying a particular mechanism of the external action, we emphasize a basic feature of the response problem in systems without external leads. A steady-state regime in such systems may be reached only due to a balance between an incoming energy flux, pumped to the system, and an outgoing energy flux to a surrounding thermostat. Therefore, the mechanism of the energy dissipation in systems without leads is vitally important and should be explicitly taken into account; the response problem becomes a kinetical one. This is in contrast to systems with leads where the energy balance may be provided by outgoing electrons and the system allows for a dynamical description (see discussion in[13]).

Below we shall consider a kinetical problem of the system response to an external field with taking into account energy dissipation processes. We assume that the "cooling" of electrons is provided by interaction with phonons which transfer efficiently the excess energy to the surrounding thermostat (note that the electron-electron in-

teraction itself does not change the total energy of the electron system). As to the field that excites the electron system, the analysis we present below may be applied for various physical mechanisms.

In particular, in order to keep the connection with our previous studies[8,10,13,15], we shall consider the action of an external electromagnetic field with the component $\mathcal{E}(t) = -(1/c)(d/dt)A(t)$ along the circumference. Such a field may arise, for instance, if the magnetic flux contains a time dependent part. We represent the vector potential $A(t)$ in the form:

$$A(t) = \int_0^\infty d\omega [A(\omega) \exp(-i\omega t) + c.c] \tag{1}$$

and assume that this external field has the Gaussian statistics with the spectral correlation function $S_A(\omega)$ determined by

$$< A(\omega) A^*(\omega') >= S_A(\omega)\delta(\omega - \omega'). \tag{2}$$

In case of a narrow spectral width (quasi-monochromatic field) the above expressions reduce to $A(\omega) = A_0\delta(\omega - \omega_0)$ and $S_A(\omega) = |A_0|^2\delta(\omega - \omega_0)$, respectively.

We are interested in the response to the perturbation

$$V(t) = -(e/c)A(t) \int d\mathbf{r} \Psi^\dagger(\mathbf{r}) \hat{\mathbf{v}} \Psi(\mathbf{r}) \,, \tag{3}$$

where $\Psi^\dagger(\mathbf{r})$ is the electron annihilation operator and $\hat{\mathbf{v}} = -i\nabla/m - eA/c$ is the electron velocity operator (m is the electron mass). More specifically, we are interested in a field-induced electron current of zero frequency (i.e. a direct current) flowing along the ring.

In addition to this particular mechanism of the external action, we shall analyze also a more general situation, assuming only that due to some "external perturbation" there arises a non-equilibrium steady-state in electron-phonon system of the sample, and we shall study how this influences the direct current. A physical origin of the perturbation may be also purely "intrinsic", caused for instance by the energy release in course of a slow relaxation of the non-equilibrium nuclear system of the sample. In this connection we also mention the slow ortho-para conversion of the H_2 molecules inside a metal. The corresponding heat release of 1 *ppm* of H_2 in Cu at 4 K could be as large as 5×10^{-3} nW/g even weeks after a cooling-down. The similar heat release is observed from the quartz glass[16].

B. Direct current, general expressions

First we shall discuss general properties of the direct current in the Aharonov-Bohm geometry. We shall use the Keldysh formalism which allows to take simultaneously into account interactions with external and internal fields (see, e.g., a review[17]). The current flowing along the ring is given by

$$I(t) = -i\frac{e}{2L}\mathbf{Tr}\{\hat{v}_x G^K(t,t)\}, \tag{4}$$

where $e = -|e|$ is the electron charge, $\hat{v}_x = -i(\nabla_x - eA/c)/m$ is the electron velocity operator (x-axis is chosen along the circumference), and $G^K \equiv (\underline{\hat{G}})_{12}$ is the Keldysh component of the Keldysh matrix Green's function in the standard triangular representation[17], and the trace is taken over spin and space coordinates. In a steady state of the electron system in the absence of interaction and time dependent external fields, the Fourier transform of $G^K(t-t')$ is given by

$$G_0^K(E) = f_E[G_0^R(E) - G_0^A(E)] \,, \tag{5}$$

where $G_0^{R(A)}(E)$ is the retarded (advanced) Green's function, and $f_E = 1 - 2n_F(E)$ is connected with the electron energy distribution function $n_F(E)$. If the considered steady state corresponds to the thermal equilibrium with temperature T, $f_E = \tanh E/(2T)$. An equilibrium persistent current for a non-interacting electron system is given by

$$I_0 = \frac{e}{L}\int \frac{dE}{4\pi i} f_E \mathbf{Tr}\{\hat{v}_x \left[G_0^R(E) - G_0^A(E)\right]\}. \tag{6}$$

In the presence of an interaction or an external perturbation, we have instead of Eq.(5)

$$G^K(E) = \left(\underline{\hat{G}}_0 + \underline{\hat{G}}_0 \underline{\hat{\Sigma}} \underline{\hat{G}}_0 + ...\right)_{12} \tag{7}$$

where $\underline{\hat{\Sigma}}$ is a self-energy matrix. In the minimal (second) order of the perturbation theory, the self-energy splits into independent parts $\underline{\hat{\Sigma}} = \underline{\hat{\Sigma}}_f + \underline{\hat{\Sigma}}_{e-ph} + \underline{\hat{\Sigma}}_{e-e}$, which correspond to different physical perturbation mechanisms (an external field, electron-phonon and electron-electron interactions, respectively). Corresponding contributions to the direct current $I = \overline{I(t)}$ have very similar structure, so it is sufficient to write down explicitly only expressions for the case of electron-phonon interaction.

1. *Electron-phonon interaction*

The time-independent current may be represented in the form:

$$I = \overline{I(t)} = I_0 + I_1 + I_2 + I_3. \tag{8}$$

Here

$$I_1 = \frac{e}{4L}\int \frac{dE\, d\omega}{(2\pi)^2} \mathbf{Tr}\{\hat{v}_x \left[G^R(E)\hat{g}G^A(E-\omega)\hat{g}G^A(E) - G^R(E)\hat{g}G^R(E-\omega)\hat{g}G^A(E)\right] \Big([f_E - f_{E-\omega}]\hat{D}^K(\omega) + [f_E f_{E-\omega} - 1]\Delta\hat{D}(\omega)\Big)\}\,, \tag{9}$$

where $\hat{g}$ is an operator vertex of the electron-phonon interaction $\hat{g}\Psi^\dagger\Psi\mathbf{u}$ that will be specified later, $\mathbf{u} = \{u_\alpha\}$ is the displacement vector; $G^{R,A}$ are electron Green's functions in the absence of interactions (for simplicity,

here and below we omit the corresponding subscript "0"); $\Delta\hat{D} \equiv \hat{D}^R - \hat{D}^A$; $\hat{D}^{R(A)}$ and $\hat{D}^{K)}$ are components of the phonon matrix Green's function $\underline{\hat{D}}$: $D^K_{\alpha,\beta}(1,1') = -i < [u_\alpha(1),\, u_\beta(1')]_+ >$, etc. In the short-hand notation the integrand in Eq.(9) may be represented as $\mathbf{Tr}\{\mathbf{J}_1\}$, where $\mathbf{J}_1$ has the following structure

$$\begin{aligned} \mathbf{J}_1 &= (\mathbf{RAA} - \mathbf{RRA})\,[(f - f')D^K \\ &+ (ff' - 1)\Delta D], \end{aligned} \tag{10}$$

The parts I_2 and I_3 have the same integral representation as Eq.(9) but with different trace structures instead of Eq.(10):

$$\mathbf{J}_2 = [\mathbf{RAR}D^R + \mathbf{ARA}D^A](1 - ff')\,; \tag{11}$$

$$\begin{aligned} \mathbf{J}_3 &= \mathbf{RRR}[fD^K + (ff' - 1)D^R] \\ &- \mathbf{AAA}[fD^K - (ff' - 1)D^A]\,. \end{aligned} \tag{12}$$

Below we shall treat the phonon subsystem as a large reservoir in equilibrium. In this case

$$\hat{D}^K(\omega) = N(\omega)\left[\hat{D}^R(\omega) - \hat{D}^A(\omega)\right]\,; \tag{13}$$

$N(\omega) = 1 + 2n_B(\omega)$, where $n_B(\omega)$ is the boson (phonon) occupation number; in the thermal equilibrium the function $N(\omega) = \coth[\omega/(2T)]$. The part I_1 (9) describes the contribution to the direct current that, being averaged over disorder, is not exponentially small at high frequencies of bosons $\omega \gg E_c$, where $E_c = D/L^2$ is the Thouless energy, D is the diffusion coefficient of electrons. This is because the combinations $\mathbf{R}(E-\omega)\mathbf{A}(E)\mathbf{R}(E)$ and $\mathbf{A}(E-\omega)\mathbf{A}(E)\mathbf{R}(E)$ allow to build a zero-frequency cooperon[8]. The part I_2 (11) is a boson analog of the Ambegaokar-Eckern contribution[18] to the averaged persistent current that is exponentially small at high frequencies of bosons. Finally, the part I_3 does not contribute to the disorder averaged current but exhibits only in mesoscopic fluctuations.

A beauty of the expressions is that the "kinetic" part I_1 is identically zero in the complete equilibrium. Indeed, using (13) we can represent Eq.(10) in the form:

$$\begin{aligned} \mathbf{J}_1 &= (\mathbf{RAA} - \mathbf{RRA})\,\Delta D \times \\ &[(f - f')N(\omega) + ff' - 1], \end{aligned} \tag{14}$$

where the last square bracket vanishes in the complete equilibrium. The existence of the kinetic contribution Eq.(14) is the main and generic difference between a non-equilibrium steady state and the complete thermodynamic equilibrium.

2. Electron-electron interaction

Expressions for I_1-I_3 are determined by Eqs.(9)-(12) with substitutions: $\hat{g} \to 1$ and $\underline{D}(\omega) \to \underline{V}(\omega)$, where $\underline{V}$ is the Keldysh matrix for the screened Coulomb interaction, $V^{R(A)} = V_0/[1 - V_0\pi^{R(A)}]$ and $V^{K)} = V^R\pi^K V^A$; here V_0 is the usual (unscreened) Coulomb potential and $\underline{\pi}$ is a (matrix) polarization operator[17] (see section IV.B).

3. Interaction with an external field

The contribution to the direct current made by an external classical field does not contain a "thermodynamical" part Eq.(11). This is due to general (causal) analytical properties of the response: there may be only zero or one change of the analyticity ($R \leftrightarrow A$) between the beginning and end points of the electron lines (see e.g.[13] and references therein). Expressions for the field-induced direct current are given by Eqs.(9), (10), and (12) with the following modifications: $\hat{g} \to -e\hat{v}_x/c$,

$$\hat{D}^K(\omega) \to -2\pi i S_A(\omega)\,, \tag{15}$$

and omission of terms proportional to $(ff' - 1)D^{R(A)}$.

4. External field vs. non-equilibrium nodes

One can easily see from Eqs.(13) and (15) that the effect of an external field is similar to the action of a non-equilibrium part of the phonon field described by a non-equilibrium component of the phonon energy distribution function $N_{neq}(\omega) = N(\omega) - \coth[\omega/(2T)]$. We arrive at an important conclusion: the absence of a complete equilibrium in the electron-phonon system of a mesoscopic ring may result in a non-equilibrium direct current similar to the DC induced by an external classical field.

In the equilibrium, the sum of $I_2 + I_3$ gives the quantity σ_{eq} from Ref.[14] (if we use the relationship $D^R(\omega) = -D^A(\omega)$ used in Ref.[14]).

III. NON-EQUILIBRIUM ENERGY DISTRIBUTION IN A PUMPED ELECTRON SYSTEM

A. Kinetic equation

From Dyson equations $\left(\underline{\hat{G}}_0^{-1} - \underline{\hat{\Sigma}}\right)\underline{\hat{G}} = \hat{I}$ and $\underline{\hat{G}}\left(\underline{\hat{G}}_0^{-1} - \underline{\hat{\Sigma}}\right) = \hat{I}$ for the matrix Green's function it follows

$$\left[\underline{\hat{G}}_0^{-1},\, \underline{\hat{G}}\right] = \left[\underline{\hat{\Sigma}},\, \underline{\hat{G}}\right]\,, \tag{16}$$

where $\underline{\hat{G}}_0^{-1}(1,1') = [i\partial_{t_1} - H(\mathbf{r}_1)]\delta(1-1')$. Taking coordinate trace from the Keldysh component of the above equation we obtain:

$$2\pi\nu\mathcal{V}\partial_t f_E(t) = \mathbf{Tr}\{\Delta\Sigma G^K - \Sigma^K\Delta G\}\,, \tag{17}$$

where ν is the averaged electron density of states at the Fermi-level, $\mathcal{V}$ is the sample volume, $\Delta\Sigma = \Sigma^R - \Sigma^A$, and the averaging over disorder realizations is implied on the right hand side. Using the second order expression for $\underline{\hat{\Sigma}}$ we arrive at the kinetic equation in the standard form

$$\partial_t f_E(t) = \mathcal{I}_f + \mathcal{I}_{e-ph} + \mathcal{I}_{e-e}\,. \tag{18}$$

Here the first term on the right hand side describes pumping by the external electromagnetic field

$$\mathcal{I}_f = \frac{1}{2\tau_f}\int_0^\infty d\omega \tilde{S}_A(\omega) [f_{E+\omega} + f_{E-\omega} - 2f_E], \quad (19)$$

where

$$\tilde{S}_A(\omega) = S_A(\omega)/\int_0^\infty S_A(\omega)d\omega \quad (20)$$

is a normalized spectral distribution of the external field and

$$\frac{1}{\tau_f} = 2D(e/c)^2\overline{A^2(t)} = 4D(e/c)^2\int_0^\infty S(\omega)d\omega \quad (21)$$

is a field induced dephasing[12].

The second, electron-phonon collision term in Eq.(18) is given by:

$$\mathcal{I}_{e-ph} = \frac{i}{4\pi\nu\mathcal{V}}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int d\mathbf{r}d\mathbf{r}' \{\Delta G(E)_{\mathbf{r},\mathbf{r}'}[\hat{g}_\alpha \Delta G(E-\omega)\Delta D_{\alpha,\beta}(\omega)\hat{g}_\beta]_{\mathbf{r}',\mathbf{r}}\} (N(\omega)[f_E - f_{E-\omega}] + f_E f_{E-\omega} - 1) \quad (22)$$

1. Model of the electron-phonon interaction

We assume that the mesoscopic ring is embedded into a medium with a high thermal conductivity. This allows us to consider the phonon subsystem of the ring as an equilibrium thermal bath. To simplify the problem, here we do not take into account effects of phonon scattering at the interface, the presence of surface (interface) phonons, etc. The electron interaction with long-wavelength crystal lattice displacements (acoustic phonons) is treated in the "jelly model" and is determined by the standard deformation potential interaction

$$H_{e-ph}^{def} = \frac{iC}{\sqrt{\mathcal{V}}}\sum_{\mathbf{p},\mathbf{q}} c^\dagger(\mathbf{p}+\mathbf{q})c(\mathbf{p})(\mathbf{q}\mathbf{u}_\mathbf{q}), \quad (23)$$

where the deformation potential $C = p_F v_F/3$, $c^\dagger$ and c are the electron creation and annihilation operators, and $\mathbf{u}_\mathbf{q}$ is the Fourier transform

$$\mathbf{u}_\mathbf{q} = \frac{1}{\sqrt{\mathcal{V}}}\int d\mathbf{r}\exp(-i\mathbf{q}\mathbf{r})\mathbf{u}(\mathbf{r}). \quad (24)$$

In systems with electron scattering by impurities (i.e., in real metals) one has to take into account also the effect of phonon-induced impurity displacements (see Refs.[19,20]). This results in an additional part of the electron-phonon interaction that does not have free parameters once the electron-impurity interaction is chosen

$$H_{e-ph}^{imp} = -\frac{i}{\sqrt{\mathcal{V}}}\sum_{\mathbf{p},\mathbf{k},\mathbf{q}} U(\mathbf{k})c^\dagger(\mathbf{p}+\mathbf{q}+\mathbf{k})c(\mathbf{p})(\mathbf{k}\mathbf{u}_\mathbf{q}). \quad (25)$$

Here

$$U(\mathbf{k}) = (1/\mathcal{V})\sum_a U_a(\mathbf{k})\exp(-i\mathbf{k}\mathbf{r}_a), \quad (26)$$

where

$$U_a(\mathbf{k}) = \int d\mathbf{r}U_a(\mathbf{r})\exp(-i\mathbf{k}\mathbf{r}) \quad (27)$$

is the Fourier transform of the potential of the a-th impurity. For the sake of completeness we write down also an expansion of the displacement $\mathbf{u}(\mathbf{r})$ over the phonon creation and annihilation operators

$$u_\alpha(r,t) = \frac{1}{\sqrt{\rho_m\mathcal{V}}}\sum_{\mathbf{q},j} e_\alpha^{(j)}\sqrt{\frac{1}{2\omega_j(q)}} (a_j(q)\exp[i(\mathbf{q}\mathbf{r}-\omega_j(q)t)] + \text{H.c.}) \quad (28)$$

and the expression for the phonon Green's function

$$\Delta D_{\alpha,\beta}(1,1') = \frac{1}{\mathcal{V}}\sum_{\mathbf{q},j}\int\frac{d\omega}{2\pi}\Delta D_{\alpha,\beta}^{(j)}(\mathbf{q},\omega)\exp[i(\mathbf{q}\mathbf{r}-\omega t)]; \quad (29)$$

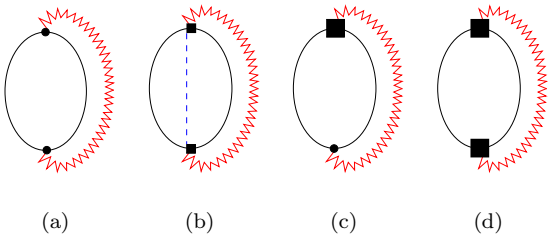
$$\Delta D_{\alpha,\beta}^{(j)}(\mathbf{q},\omega) = -\frac{\pi}{\rho_m\omega_j(q)}\times [\delta(\omega-\omega_j(q)) - \delta(\omega+\omega_j(q))]\,\eta_{\alpha,\beta}^{(j)}(\mathbf{q}). \quad (30)$$

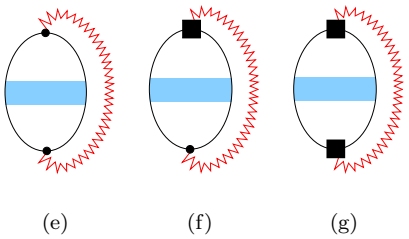
In Eqs.(28)-(30) ρ_m is the material density; j accounts for the longitudinal ($j = l$) and two (equivalent) transverse phonon polarizations ($j = t_1$ and $j = t_2$). For longitudinal phonons $\eta_{\alpha,\beta}^{(l)}(\mathbf{q}) = q_\alpha q_\beta/q^2$; while for transverse phonons

$$\sum_{j=t_1,t_2}\eta_{\alpha,\beta}^{(j)} = \delta_{\alpha,\beta} - q_\alpha q_\beta/q^2. \quad (31)$$

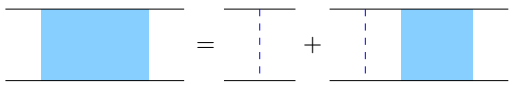
To perform the disorder averaging of the expression $\mathbf{Tr}\{\Delta G(E)\hat{g}_\alpha\Delta G(E-\omega)\hat{g}_\beta\}$ in the collision integral (22), one should take into account that the impurity potential U enters also the interaction vertex $g_\alpha^{imp} = -iU(\mathbf{k})k_\alpha$, that corresponds to Eq.(25). Various contributions to the averaged value of $\mathbf{Tr}\{\Delta G(E)\hat{g}_\alpha\Delta G(E-\omega)\hat{g}_\beta\}$ are represented by diagrams shown in Fig.1 (they are similar to those describing a disorder averaged electron self-energy in the paper by Reizer and Sergeev[20]).

First we consider the contribution of transverse acoustic phonons. The deformation part Eq.(23) of the electron-phonon interaction is absent in this case, as well as the diagrams a, c, e, and f of Fig.1. Moreover, there is no effect of "diffuson dressing" of interaction vertices (the diagram g): a dressed vertex with an entering phonon wave vector $\mathbf{q}$ is proportional to $\mathbf{q}$ and does not contribute due to the transverse structure Eq.(31) of the phonon Green's function. Thus, the leading contribution





(h) An effective (renormalized) vertices of the electron-phonon-impurity interaction.



(i) Diffuson proparators.

FIG. 1: (a-g) - diagrams for the averaged kernel of the electron-phonon collision integral in the kinetic equations. Lines describe electron propagators; dashed and zigzag lines correspond to the correlation function of the disorder potential and to the phonon propagator, respectively. Circles and small squares denote vertices of the usual (deformation potential) electron-phonon interaction and of the electron-phonon-impurity interaction, respectively; large squares denote an effective (renormalized) vertices of the electron-phonon-impurity interaction, determined by the diagram h.

is given by simple ("Drude") diagrams (b and d) and one obtains the following expression for the $\mathcal{I}_{e-ph}$ (22)

$$\mathcal{I}_{e-ph} = \int_{-\infty}^{\infty} d\omega K(\omega) \left(N(\omega)[f_{E+\omega} + f_{E-\omega} - 2f_E] + f_E[f_{E+\omega} - f_{E-\omega}] + 2\right) . \quad (32)$$

Here the kernel $K(\omega)$ is given by

$$K(\omega) = \frac{3\beta_t \omega}{2p_F^2 l v_t} \Phi_t\left(\frac{|\omega| l}{v_t}\right) , \quad (33)$$

where v_t is the sound velocity of transverse acoustic phonons and a dimensionless (transverse) coupling constant β_t is determined as

$$\beta_t = \left(\frac{p_F v_F}{3}\right)^2 \frac{\nu}{\rho_m v_t^2} \quad (34)$$

and the function $\Phi_t(x)$ is defined by

$$\Phi_t(x) = 1 + 3[x - (x^2+1)\mathrm{arctan}(x)]/(2x^3). \quad (35)$$

In the limiting cases: $\Phi_t(x) \approx x^2/5$ at $x \ll 1$ and $\Phi_t(x) \approx 1$ at $x \gg 1$. The same function arises in studies of phonon effects on the conductivity of disordered metals[20,21].

Similarly, one may obtain a contribution of longitudinal phonons. Now all the diagrams a-g in Fig.1 should be taken into account. In the limit of small phonon momenta $q \ll 1/l$, each of the diagrams (e, f, and g) with the diffuson installation is considerably larger than the corresponding generic diagram (a, c, and d, respectively). However, in the leading order in $1/(ql)^2$ the diffuson diagrams mutually cancel and their remaining parts are comparable with contributions made by diagrams a-d. Such a suppression of the diffuson dressing of electron-phonon interaction vertices, demonstrated by Reizer and Sergeev[20], occurs due to a destructive interference of contributions made by the usual deformation potential Eq.(23) and by its electron-phonon-impurity counterpart Eq.(25): the diagrams e and f are canceled by the cross-terms described by the diagram f and its symmetric partner (not shown). The physical mechanism of such a cancellation is connected with the Galilean transformation from the laboratory system to a system moving together with the crystal lattice[19,20].

The resulting expressions for the contribution of longitudinal phonons differ from Eq.(33)-(35) by the replacement $\beta_t \to \beta_l$ (where $v_t \to v_l$) and $\Phi_t(x) \to \Phi_l(x)$, where $\Phi_l(x) \sim x^2$ at $x \ll 1$. Typically, v_t is by factor 2-3 smaller than v_l, that allows one to neglect safely the contribution of longitudinal phonons[21].

B. Steady state solution to kinetic equation

We are interested in a steady-state regime of the electron subsystem subjected to a stationary external pumping. The only collision channel that takes off the injected energy, is the electron-phonon interaction. The electron-electron interaction conserves the total energy but leads to a redistribution of electrons over energy levels. In a closed electron system with a given energy this would effectively lead to a quasi-equilibrium Fermi distribution with some effective temperature T_e. We shall assume that this process of establishing a quasi-thermal distribution in the electron system is faster than the cooling rate due to the electron-phonon collisions (similar approach was used in Ref.[22] in studying non-equilibrium currents in a metal ring). This allows us to use an adiabatic approach: we put $f_E(t) = \tanh[E/(2T_e(t))]$ and $N(E) = \coth[E/(2T)]$ into Eq.(18) and integrate over E with the weight E. Due to the energy conservation, the electron-electron collision term makes no contribution and we arrive at a closed equation for the effective

electron temperature T_e:

$$\frac{4\pi^2 T_e}{3}\frac{dT_e}{dt} = \frac{\omega_0^2}{\tau_f} + 4\int_0^\infty d\omega\,\omega^2 K(\omega)\left[\coth\left(\frac{\omega}{2T}\right) - \coth\left(\frac{\omega}{2T_e}\right)\right]. \quad (36)$$

where ω_0 is the "mean-square-root" of the external field frequency

$$\omega_0^2 \equiv \int_0^\infty \omega^2 \tilde{S}(\omega)\, d\omega\,. \quad (37)$$

In the derivation of Eq.(36) we have used an identity $ff' - 1 = -N(E - E')(f - f')$ for the Bose and Fermi distribution functions at the same temperature.

An effective electron temperature T_e in a steady-state regime is determined by the stationary solution to Eq.(36). Small deviations δT_e from the stationary solution T_e are described by a linearized Eq.(36):

$$d\delta T_e/dt = -\delta T_e/\tau_T, \quad (38)$$

where the temperature relaxation rate is given by

$$\frac{1}{\tau_T} = \frac{3}{2\pi^2 T_e^3}\int_0^\infty \frac{\omega^3 K(\omega)}{\sinh^2[(\omega/(2T_e)]} d\omega\,. \quad (39)$$

To obtain explicit expressions for T_e and τ_T, we restrict our consideration to the limiting cases where the electron elastic mean-free path l is small or large as compared to the phonon wavelength $\sim v_t/\omega$ which corresponds to typical transfer energies $\omega \sim T_e$.

Using Eqs.(33) and (35) in the case of *small* $x = \omega l/v_t$, we arrive at the following expression for the effective steady-state electron temperature T_e

$$T_e = \left[T^6 + \frac{\hbar^4\omega_0^2 p_F^2 v_t^3}{288\zeta(6)\beta_t k_B^6 l\tau_f}\right]^{1/6} \quad (40)$$

and for the temperature relaxation rate $1/\tau_T$:

$$\frac{1}{\tau_T} = \frac{54\zeta(6)\beta_t l(k_B T_e)^4}{5\pi^2\hbar^2 p_F^2 v_t^3}\,. \quad (41)$$

In these expressions $\zeta(x)$ is the Riemann zeta-function ($\zeta(6) \approx 1$); for further estimates we have written explicitly the Planck and Boltzmann constants, $\hbar$ and k_B. As we see from Eq.(40), a noticeable overheating takes place at intensities of the external field determined by Eq.(21) and the condition

$$\frac{1}{\tau_f} \sim \frac{3\cdot 10^2\beta_t l(k_B T)^6}{\hbar^4\omega_0^2 p_F^2 v_t^3}\,. \quad (42)$$

At relatively high temperatures T or at a considerable overheating ($T_e \gg T$), the assumption of small ratio $x = \omega l/v_t \sim T_e l/v_t$ may be violated. For the opposite case of relatively *large* transfer energy $x = \omega l/v_t \sim T_e l/v_t \gg 1$ the kernel $K(\omega) \sim \omega$ and we find:

$$T_e = \left[T^4 + \frac{\hbar^2\omega_0^2 p_F^2 v_t l}{72\zeta(4)\beta_t k_B^4\tau_f}\right]^{1/4} \quad (43)$$

and

$$\frac{1}{\tau_T} = \frac{216\zeta(4)\beta_t(k_B T_e)^2}{\pi^2 p_F^2 v_t l}\,. \quad (44)$$

For the validity of the used adiabatic approach we should require the smallness of the temperature relaxation rate $1/\tau_T$ as compared to the rate $1/\tau_{e-e}$ of establishing a quasi-thermal Fermi distribution due to electron-electron collisions. An estimate for τ_{e-e} follows from the electron-electron collision term[17] in the considered quasi-1D geometry:

$$\frac{1}{\tau_{e-e}} \sim \frac{k_B T_e}{\hbar g}\sqrt{E_c/E_{cut}}\,. \quad (45)$$

Here $g = E_c/\Delta$ is the dimensionless conductance ($\Delta = 1/(\nu\mathcal{V})$) is the mean electron energy level spacing), E_{cut} is a typical low energy cutoff that depends on the relationship between E_c and T_e, but at any case $E_{cut}\tau \ll 1$, where $\tau = l/v_F$ is a mean free time for elastic scattering. Comparing the electron "thermalization" rate $1/\tau_{e-e}$ with the temperature relaxation rate $1/\tau_T$ Eq.(41) (considering, for instance, the case of small typical phonon momenta $k_B T_e/(\hbar v_t) < 1/l$), we obtain:

$$\frac{\tau_{e-e}}{\tau_T} \sim \frac{S}{l^2}\left(\frac{k_B T_e l}{\hbar v_t}\right)^3\sqrt{E_{cut}\tau}\,. \quad (46)$$

For typical experimental configurations the ring cross section S is comparable with l^2, and the ratio τ_{e-e}/τ_T is small due to two other factors on the right hand side of Eq.(46). This justifies the validity of the adiabatic approach.

C. Numerical estimates

To get an idea about an overheating effect, consider a gold ring. Following ref.[21], we take the following material parameters: $\beta = 1.4$, $p_F = 1.3\cdot 10^{-19}\mathrm{g\cdot cm/s}$, and $v_t = 1.2\cdot 10^5\mathrm{cm/s}$. Then Eq.(40) may be represented in the form

$$T_e \approx 70\mathrm{mK}\frac{(\omega_0\cdot 10^{-9}\mathrm{s})^{1/3}}{[(l\cdot 10^5\mathrm{cm}^{-1})(\tau_f\cdot 10^8\mathrm{s}^{-1})]^{1/6}} \quad (47)$$

where it has been assumed that $T_e \gg T$. Eq.(40) has been derived under the assumption of small transferred momenta $T_e/v_t \lesssim 1/l$, that may be represented as

$$T_e \lesssim 90\mathrm{mK}/(l\cdot 10^5\mathrm{cm}^{-1}). \quad (48)$$

If the fraction on the right hand side of Eq.(47) does not exceed unity, the expressions (47) and (48) are compatible.

We see, that the presence of a high frequency external pump field may result in a noticeable overheating of the electron system. For $\omega_0 \sim 10^9 \mathrm{s}^{-1}$ and $l \sim 10^{-5}$cm, the electron temperature T_e is measured by tens mK even for such an external field that leads to a quite moderate field-induced dephasing rate on a scale of 10^7-$10^8 \mathrm{s}^{-1}$. We note also that T_e depends very weakly (as $P^{1/6}$) on the pumping power P.

The overheating may put serious obstacles in investigations of coherent nonlinear phenomena in closed mesoscopic samples.

IV. NON-EQUILIBRIUM EFFECTS ON THE DIRECT CURRENT

As discussed in the preceding section, pumping the electron system may result in a noticeable deviation of the electron distribution function from the equilibrium one. The absence of the complete equilibrium between the electron and phonon subsystems may lead to an additional contribution to the averaged direct current. This contribution cannot be considered in the framework of a simplified dynamical treatment used in the earlier work[8] (see also[13]), it requires the kinetic approach. Here we shall study this effect. In the first and second subsections we shall consider non-equilibrium phonon and Coulomb contributions to disorder averaged DC. Of principal importance is the non-equilibrium contribution connected with the part I_1 Eq.(9) of the DC, as this part vanishes in the equilibrium. Calculation of disorder averaged quantities is performed within the usual formalism of weak localization theory. The only point to be commented is the appearance of a diffuson carrying zero frequency and wave vector. This singular diffuson ("loose" diffuson[13]) is a typical feature of non-equilibrium problems in mesoscopic systems. Such a diffuson may be built by drawing parallel disorder lines embracing the "self-energy" part between R and A Green's function (see Fig.3 in Appendix). The role of the anomalous diffuson has been studied earlier[13] in the framework of the dynamical consideration and it is closely connected with the renormalization of the electron energy distribution function. Here (see Appendix) we prove an important identity for the anomalous diffuson. Namely, we show that when taking into account all the mechanisms contributing to the "self-energy", the diagrams with the anomalous diffuson cancel each other at a *steady-state* regime. Thus the loose diffusons signal on the incorrect or time-dependent energy distribution function, and they cancel out as soon as the correct steady-state distribution function f_E is substituted in Eq.(5).

A. Non-equilibrium electron-phonon interaction effects on DC

The question about a DC induced due to the absence of the equilibrium between electrons and phonons, has been put forward by Spivak, *et al.*[22], who have calculated the DC variance. Here we shall consider not the variance but the disorder averaged DC (in the presence of a magnetic flux pierced the ring). As in the preceding sections we restrict our consideration to the case of transverse acoustic phonons.

The leading contribution to DC is given by a diagram (see Fig.2) containing a loop of two cooperon propagators connecting two "Hikami boxes"(a rhomb and a triangle).

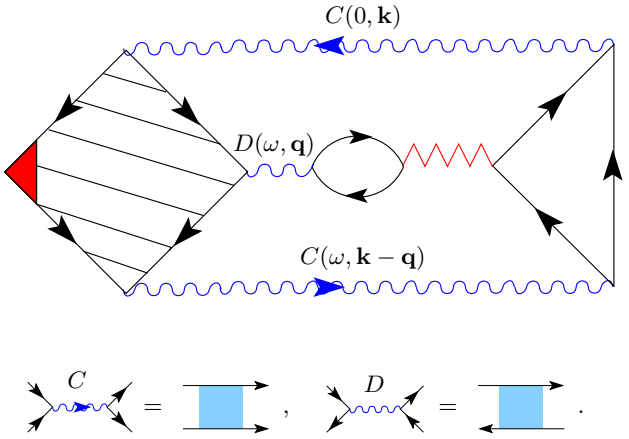


FIG. 2: A leading contribution of the electron-phonon or electron-electron interaction to a non-equilibrium part of the time-independent (DC) current. Wavy lines correspond to cooperons (C) or diffusons (D); the zigzag line corresponds to the screened electron-electron interaction or the phonon propagator. In the case of longitudinal phonons both electron-phonon and renormalized electron-phonon-impurity vertices should be taken into account. For the case of transverse phonons the diffuson should be omitted, and the rhomb and triangle are directly connected by the phonon propagator (the zigzag line) terminated by the renormalized electron-phonon-impurity vertices.

One of the cooperons carries zero frequency and a small wave vector $k = 2\pi(n-2\phi)/L$ along the ring circumference (i.e.,"x"-axis); ϕ is a magnetic flux measured in units of $hc/|e|$. It is the cooperon that provides the dependence of the effect on the magnetic flux. The dependence of the second cooperon $C(k-q,\omega)$ on the magnetic flux is negligible at phonon wave vectors $q >> 1/L$. Expressions for the Hikami boxes are given by $F_\alpha(q)$ for the rhomb and by $kF_\beta(q)$ for the triangle. The vector function $F_\alpha(q)$ is defined (for the RRA succession of Green's functions) as

$$F_\alpha(q) = \delta_{\alpha,x}\frac{p_F l}{3}\frac{1}{V}\sum_p G^R(p+q)G^A(p) + \frac{i}{V}\sum_p v_x p_\alpha G^R(p+q)[G^A(p)]^2 \qquad (49)$$

For both RRA and RAA cases, the corresponding functions F_α coincide and equal to

$$F_\alpha = \delta_{\alpha,x}\frac{2\pi\nu\tau p_F l}{3}h(ql) \tag{50}$$

(a longitudinal part $\sim q_\alpha$ has been omitted here), where

$$h(x) = \frac{\arctan(x)}{x} + \frac{3[\arctan(x) - x]}{x^3} \tag{51}$$

At large x, $h(x) \approx \pi/(2x)$. At small x, $h(x) \approx (4/15)x^2$. For m-th harmonic of the phonon induced averaged current we obtain

$$I_1^{(m)} = \frac{1}{6\pi^3}\beta_t\frac{e}{\nu D u_t}\int_0^\infty d\omega\,\omega^2 h^2(\omega l/u_t) \\ [\coth(\omega/(2T)) - \coth(\omega/(2T_e))]\mathrm{Im}C(\omega/u_t,\omega)\,. \tag{52}$$

The range of small momentum transfer $q = \omega l/u_t \ll 1$ corresponds to the diffusion regime, where $\mathrm{Im}C(q,\omega) = \omega/(\omega^2 + D^2q^4)$. In this case the leading contribution to the integral stems from the range $u_t^2/(v_F l) \ll \omega \ll u_t/l$ and we obtain

$$I_1^{(m)} = \frac{\zeta(4)}{\pi^3}\left(\frac{4}{15}\right)^2\beta_t\frac{el^4(T_e^4 - T^4)}{\nu D^3 u_t} \tag{53}$$

Therefore we have an estimate (for $T_e - T \gtrsim T$):

$$I_1^{(m)} = eE_c[(T_e/E_c)(\tau T_e)(T_e/E_F)(T_e/\omega_D)], \tag{54}$$

where $\omega_D \sim u_t p_F$ is the Debye frequency. The factor eE_c is a typical (mesoscopic) magnitude of a persistent current in a single ring, but the expression in the square brackets is extremely small.

Even smaller quantity arises in the case of large momentum transfer $q = \omega l/u_t \gg 1$ in Eq.(52), that may take place at relatively large T_e, i.e. $T_e > u_t/l$, the latter quantity is of the order of 100 mK for $u_t = 10^5$cm/s and $l = 10^{-5}$cm. In this case we shall use the ballistic expression for the "cooperon", that reads: $\mathrm{Im}C(q,\omega) = \omega/(v_F q)^2$ (at $\omega \ll v_F q$). As a result, we shall obtain an expression which is by a factor $[u_t/(lT_e)]^3 \ll 1$ smaller than Eq.(54).

We arrive at the conclusion that the non-equilibrium contribution of electron-phonon interaction to the flux-dependent part of the averaged DC is negligible. This is compatible with the smallness of the photovoltaic current variance found by Spivak, *et al.*[22].

B. Non-equilibrium electron-electron interaction effects on DC

As noted in sec.II, effects of the absence of the complete equilibrium in pumped electron systems may result in a non-zero contribution I_1 to DC. This effect takes place for the phonon contribution to I_1 studied in the preceding section, although the magnitude of the effect turns out to be small. Considering the contribution of the electron-electron interaction to I_1 we deal with the modification $\underline{\hat{D}}(\omega) \to \underline{V}(\omega)$ of the (symbolic) expression $\mathbf{J}_1$ Eq.(10) (see subsection IIB1):

$$\mathbf{J}_1 = (\mathbf{RAA} - \mathbf{RRA})\,[(f - f')V^K \\ + (ff' - 1)\Delta V] \tag{55}$$

Here $V^{R(A)} = V_0/[1 - V_0\pi^{R(A)}]$ and $V^{K)} = V^R\pi^K V^A$ are elements of the Keldysh matrix $\underline{V}$ for the screened Coulomb interaction; $\Delta V = V^R - V^A$; V_0 is the usual (unscreened) Coulomb potential; and $\underline{\pi}$ is a (matrix) polarization operator[17]. In dirty metals (in the diffusive electron propagation regime) matrix components of the screened Coulomb "potential" are given by (see[17,23]):

$$\Delta V(q,\omega) = -\frac{2i\omega}{Dq^2\nu}, \tag{56}$$

and

$$V^K(q,\omega) = -\frac{2i\overline{\omega}}{Dq^2\nu}, \tag{57}$$

where ν is the density of states (at the Fermi energy) corresponding to the effective sample dimension, and the quantity $\overline{\omega}$ is defined by

$$\overline{\omega} = \frac{1}{2}\int_{-\infty}^{\infty}[1 - f_E f_{E-\omega}]\,dE\,. \tag{58}$$

To obtain the contribution of the part Eq.(55) to the disorder averaged DC, we should average the triangle of electron Green's functions $(\mathbf{G^R}(\mathbf{E})\mathbf{G^A}(\mathbf{E}-\omega)\mathbf{G^A}(\mathbf{E}) - \mathbf{G^R}(\mathbf{E})\mathbf{G^R}(\mathbf{E}-\omega)\mathbf{G^A}(\mathbf{E}))$ and integrate the resulting expression over E. Formally, the leading contribution to the kinetic part $\mathbf{J_1}$ of the averaged DC would be given by the two-cooperon diagram in Fig.2. However, we will show that this contribution, as well as contributions of higher order diagrams, vanish after the integration over E. Due to the presence of distribution functions with the asymptotic properties $f_E \to \pm 1$ at $E \to \pm\infty$, the integration over E runs in a close vicinity of the Fermi energy. Here we restrict ourselves to the usual approximation of the weak localization theory, namely, we neglect a weak dependence of averaged products of electron Green's functions on the common part E of their arguments; we neglect also the energy dependence of the averaged density of states (DOS). In this approximation the only E-dependent part of the integrand is given by the square bracket $[\,...\,]$ in Eq.(55) and we find:

$$\int_{-\infty}^{\infty}[\,...\,]\,dE = 2\omega V^K(q,\omega) - 2\overline{\omega}\Delta V(q,\omega) = 0. \tag{59}$$

(the latter equality follows from Eqs.(56) and (57)). This result has been obtained without any assumption about the form of the distribution function f_E; the derivation is based on the identity

$$\int_{-\infty}^{\infty}[f_E - f_{E-\omega}]dE = 2\omega \tag{60}$$

valid for an *arbitrary* distribution function f_E (satisfying the standard asymptotic conditions). We arrive at the conclusion that effects of non-equilibrium and non-thermal electron distribution do not contribute to the "kinetic" part I_1 of the direct current. Of course, there is always a "thermodynamic" part I_2 of the direct current, considered by Ambegaokar and Eckern[18], but this part is nonzero even in the complete equilibrium and is only weakly changed by a relatively weak pumping.

It should be emphasized once again that the vanishing of the I_1 part of the electron-electron contribution to DC has been obtained in the approximation of the energy-independent DOS and the averaged electron triangle, as well as neglecting disorder-induced correlations between the "electron triangle" and the polarization operator. Results obtained beyond the constant DOS approximation will be reported elsewhere[24].

V. CONCLUSIONS

We have considered kinetics of closed mesoscopic samples (metal rings) subjected to an external ac pump field. To provide a consistent description of excitation and relaxation processes in the pumped electron system, electron-phonon interaction has to be taken explicitly into account. With the use of Keldysh technique we have derived general expressions for a time-independent electric current along the ring in the presence of a static magnetic flux. These expressions describe simultaneously a thermodynamical persistent current (PC) and a direct current (DC) induced nonlinearly by the ac field or, in general, by an arbitrary perturbation of the equilibrium state. The kinetic approach allows to avoid a problem of anomalous (loose) diffusons[13]. The singularity represented by a loose diffuson in the particular case of diffusive mesoscopic systems, is generic to non-equilibrium systems described in the framework of the Keldysh technique. The origin of this singularity is the ansatz Eq.(5). The singularity arises if the energy distribution function f_E does not satisfy the kinetic equation (e.g., if it is chosen to be the equilibrium Fermi distribution). We show that the anomalous diagrams with loose diffusons vanish in the steady state regime if f_E is chosen to make the inelastic collision integral vanish.

Solving the kinetic equation for the electron system, pumped by an external field and cooled through the phonon bath, we have derived expressions for an effective electron temperature T_e. We have found that for appreciable external field intensities, there may be a noticeable overheating of the electron system with T_e being on the scale of tens mK. This overheating may hinder manifestations of mesoscopic effects (like PC phenomena). The analysis restricts the range of permissible external pump intensities.

In addition to crude thermal effects, there may be more subtle phenomena. Namely, the DC may be induced not only by the external field but also due to the absence of a complete equilibrium between electron and phonon subsystems. To have a self-consistent description, we have examined such non-equilibrium contributions to DC. We have found that the phonon contribution to DC caused by the difference of phonon (T) and electron (T_e) temperatures, is negligible as compared to the typical PC magnitude. We have studied also the contribution to DC caused by the Coulomb interaction of non-equilibrium electrons, for a non-Fermi distribution of electrons over energy levels. We have found that within the framework of usual approximations of the weak localization theory (neglecting energy dependence of the electron density of states near the Fermi surface), this non-equilibrium distribution vanishes identically. This analysis generalizes, supports, and shows the limits of applicability of the results of earlier works[8,10], where only the field-induced contribution to DC was considered.

Acknowledgments

We are thankful to O.L.Chalaev for useful discussions. The work was partially supported by the grant from the Ministry of Science and Education of Japan (Mombusho), by the RFBR grant No.98-02-16062, and by the grant "Nanostructures" from the Russian Ministry of Science (V.I.Y.).

VI. APPENDIX: VANISHING OF THE ANOMALOUS DIFFUSON CONTRIBUTION

As mentioned in the text, there may be anomalous diagrams with a "loose diffusion" part[13]. Such a diffuson may be constructed of retarded (G^A) and advanced (G^R) Green's functions in- and outgoing from a self-energy installation Σ, see Fig.3.

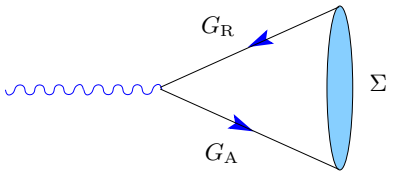


FIG. 3: A "head" of the anomalous diffuson.

As this installation conserves energy (in case of an external field we separate a part with equal numbers of absorbed and emitted "photons"), the energy arguments of G^R and G^A coincide, hence the loose diffusion carriers a zero frequency. An anomaly arises in the case, where the diffusion "head" ($G^R\Sigma G^A$) is averaged over the disorder independently of the averaging the rest part of the diagram. Due to the restoration of the translational symmetry, the averaged "head" transfers no momentum to the loose diffuson. Thus, we arrive at a singularity connected with a diffuson of zero frequency and wave vector.

Our present task is to show that the contribution of anomalous diagrams vanishes in a steady state regime. Namely, we shall argue for the vanishing of the diffuson "head" $\mathcal{H}(E)$

$$\mathcal{H}(E) = \mathbf{Tr}\,[\underline{G}(E)\underline{\Sigma}(E)\underline{G}(E)]^K_{RA}\ , \qquad (61)$$

where all functions are in Keldysh's space; the trace is performed over space coordinates; the subscript RA means that we need to keep only the part with outgoing G^R and incoming G^A lines. Using the relationship $G^K(E) = f_E \Delta G(E)$, we have:

$$\begin{aligned}\mathcal{H}(E) &= \mathbf{Tr}\left[G^R(E)[-f_E\Sigma^R(E)\right.\\ &+ \left.\Sigma^K(E) + f_E\Sigma^A(E)]G^A(E)\right]^{irr}\ . \qquad (62)\end{aligned}$$

Performing here the disorder averaging one should take into account only an "irreducible" part (this is marked by a superscript irr), i.e., those diagrams which do not include an anomalous diffuson ladder built between G^R and G^A lines, as this loose diffuson has been already extracted from the "head" (see Fig.3). First, we shall establish a useful identity for an "irreducible vertex" $\mathcal{R}^{irr}$

$$\mathcal{R}^{irr}(\mathbf{r}_1,\mathbf{r}_2;E) = \int d\mathbf{r}\,[G^R(\mathbf{r},\mathbf{r}_1;E)G^A(\mathbf{r}_2,\mathbf{r};E)]^{irr} \qquad (63)$$

formed by G^R and G^A lines connected with the same site. We shall use the following trick. For the moment, we treat an infinitesimal positive quantity δ in the definition of exact electron Green's functions $\hat{G}^{R(A)} = [E - \hat{H} \pm i\delta/2]^{-1}$ as a finite constant, with taking the limit $\delta \to +0$ afterwards. This allows to deal with a *"reducible vertex"* $\mathcal{R}$ with no restriction for the arrangement of disorder lines on averaged diagrams. This relationship between $\mathcal{R}$ and $\mathcal{R}^{irr}$ is shown symbolically in Fig.4.

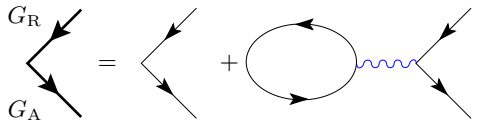


FIG. 4: Relationship between reducible (bold) and irreducible vertices.

The reducible vertex contains an anomalous part with a diffusion of zero frequency and momenta. This diffusion equals $1/\delta$. In the leading order in $\delta\tau(E) \ll 1$, where $1/\tau(E)$ is the rate of elastic scattering, we obtain

$$\mathcal{R}^{irr}(\mathbf{r}_1,\mathbf{r}_2;E) = \delta\tau(E)\mathcal{R}(\mathbf{r}_1,\mathbf{r}_2;E). \qquad (64)$$

Note that neither the left, nor the right hand sides of Eq.(64) are singular in the limit $\delta \to +0$. Representing the Green's functions in terms of exact eigenstates ψ_μ and eigenenergies E_μ

$$G^{R(A)}(\mathbf{r},\mathbf{r}';E) = \sum_\mu \frac{\psi_\mu(\mathbf{r})\psi^*_\mu(\mathbf{r}')}{E - E_\mu \pm i\delta/2}\,, \qquad (65)$$

we obtain the following expression for the reducible vertex:

$$\begin{aligned}\mathcal{R}(\mathbf{r}_1,\mathbf{r}_2;E) &= \int d\mathbf{r}\, G^R(\mathbf{r},\mathbf{r}_1;E)G^A(\mathbf{r}_2,\mathbf{r};E)\\ &= (i/\delta)\Delta G(\mathbf{r}_2,\mathbf{r}_1;E)\,, \qquad (66)\end{aligned}$$

where $\Delta G = G^R - G^A$. With the use of Eqs.(64) and (66), we arrive at the final expression for the irreducible vertex R^{irr} of our interest:

$$\mathcal{R}^{irr}(\mathbf{r}_1,\mathbf{r}_2;E) = i\tau(E)\Delta G(\mathbf{r}_2,\mathbf{r}_1;E). \qquad (67)$$

As neither the left, nor the right hand sides of Eq.(67) are singular, we may safely take the limit $\delta \to +0$. Using Eq.(67) we find for the diffuson head $\mathcal{H}(E)$

$$\begin{aligned}\mathcal{H}(E) &= -i\tau(E)\mathbf{Tr}\left[[f_E\Sigma^R(E) - \Sigma^K(E)\right.\\ &- \left.f_E\Sigma^A(E)]\Delta G(E)\right]\ . \qquad (68)\end{aligned}$$

The trace on the right hand side of the last equation coincides identically with that on the right hand side of the kinetic equation (17). We conclude that the contribution of anomalous diagrams vanishes in a steady state regime, if the correct steady-state distribution function f_E is used in the ansatz Eq.(5).

[1] M.Buttiker, Y.Imry, and R.Landauer, Phys.Lett. A **96**, 365 (1983).
[2] L.P.Levy, G.Dolan, J.Dunsmuir, and H.Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).
[3] V.Chandrasekhar, R.A.Webb, M.J.Brady, M.B.Ketchen, W.J.Gallagher, and A.Kleinsasser, Phys.Rev.Lett. **67**, 3578 (1991).
[4] P.Mohanty, E.M.Q.Jariwala, and R.A.Webb, Phys. Rev. Lett. **78**, 3366 (1997).
[5] E.M.Q.Jariwala, P.Mohanty, M.B.Ketchen, and R.A.Webb, Phys.Rev.Lett. **86**, 1594 (2001).
[6] W.Rabaud, L.Saminadayar, D.Mailly, K.Hasselbach, A.Benoit, and B.Etienne, Phys.Rev.Lett. **86**, 3124 (2001).
[7] See, e.g., review: G.Montambaux, in: *Quantum Fluctuations*, edited by S.Reynaud, E.Giacobino, and J.Zinn-Justin (Elsevier, Amsterdam, 1997).
[8] V.E.Kravtsov and V.I.Yudson, Phys.Rev.Lett. **70**, 210 (1993); A.G.Aronov and V.E.Kravtsov, Phys.Rev. B **47**, 13409 (1993); V.E.Kravtsov, Phys.Lett. A **172**, 452 (1993).
[9] P.Mohanty, Ann. Phys.(Leipzig) **8**, 7 (1999).
[10] V.E.Kravtsov and B.L.Altshuler, Phys.Rev.Lett. **84**, 3394 (2000).
[11] B.L.Altshuler, M.E.Gershenson, and I.L.Aleiner, cond-mat/9803125 (unpublished); Yu.B.Khavin,

M.E.Gershenson, and A.L.Bogdanov, Phys. Rev. Lett. **81**, 1066 (1998).
[12] B.L.Altshuler, A.G.Aronov, and D.E.Khmelnitskii, J.Phys. C **15**, 7367 (1982).
[13] V.I.Yudson, E.Kanzieper, and V.E.Kravtsov, Phys.Rev. B **64**, 045310 (2001)
[14] V.E.Kravtsov and V.I.Yudson, cond-mat/9712149 (unpublished).
[15] V.I.Yudson, Phys. Rev. B **65**, 115309 (2002).
[16] F.Pobell *Matter and methods at low temperatures*, Springer-Verlag, p.179.
[17] J.Rammer and H.Smith, Rev.Mod.Phys. **58**, 323 (1986).
[18] V.Ambegaokar and U.Eckern, Phys.Rev.Lett. **65**, 381 (1990); Europhys.Lett. **13**, 733 (1990); U.Eckern, Z.Phys. B **82**, 393 (1991); A.Schmid, Phys.Rev.Lett. **66**, 80 (1991); U.Eckern and P.Schwab, Adv. Phys. **44**, 387 (1995).
[19] A.Schmid, Z.Phys. **259**, 421 (1973).
[20] M.Yu.Reizer and A.V.Sergeev, Zh.Eksp.Teor.Fiz. **92**, 2291 (1987) [Sov.Phys.JETP **65**(6), 1291 (1987)].
[21] N.G.Ptitsina, *et al.* Phys.Rev. B **56**, 10089 (1997).
[22] B.Spivak, F.Zhou, and M.T.Beal Monod, Phys. Rev. B **51**, 13226 (1995).
[23] B.L.Altshuler and A.G.Aronov, in *Electron-Electron Interactions in Disordered Systems*, eds. A.L.Efros and M.Pollak (North-Holland; Amsterdam, Oxford, New York, Tokyo, 1985), p.1.
[24] O.L.Chalaev and V.E.Kravtsov, cond-mat/0204176 (unpublished).

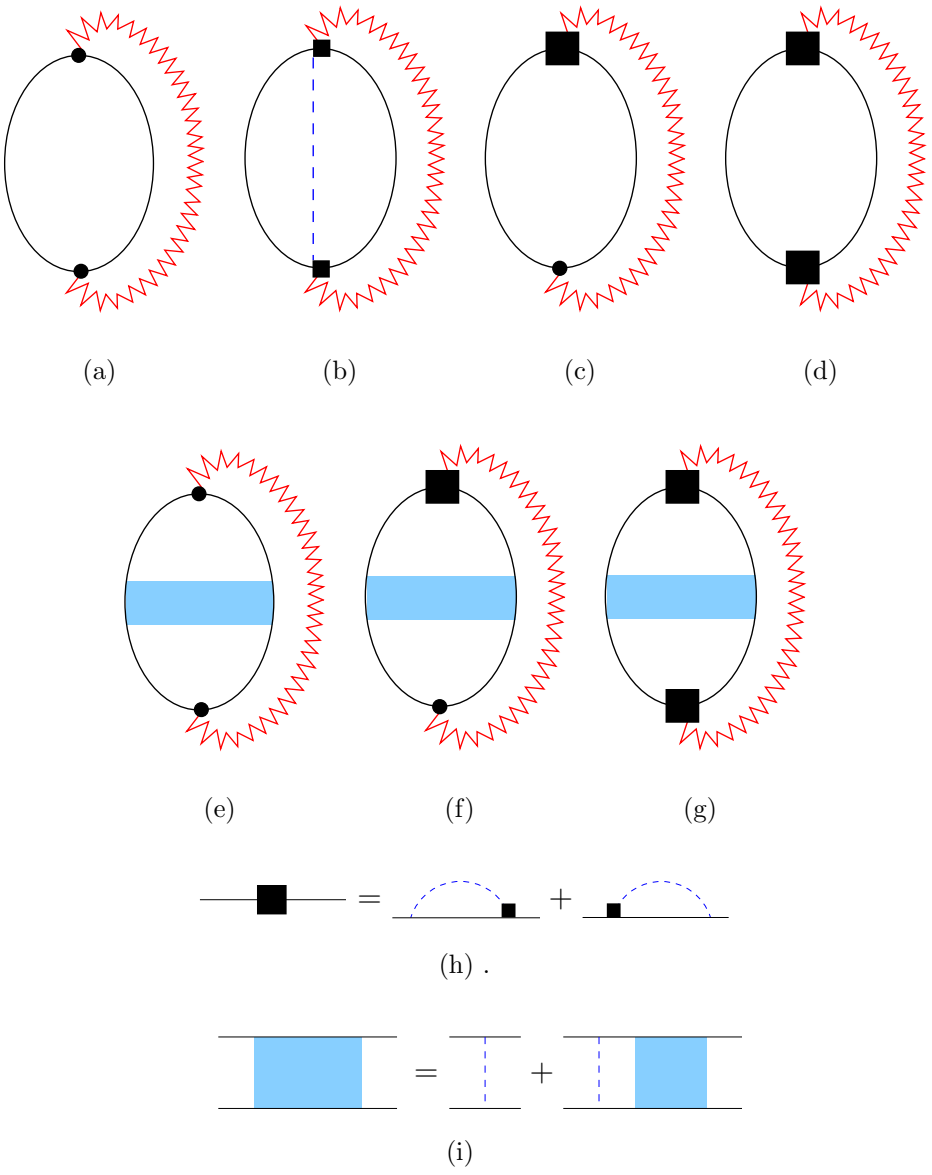


Figure 1: The general caption for this figure.

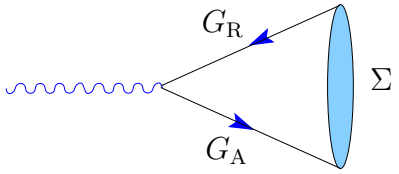


Figure 2: The caption for this figure.

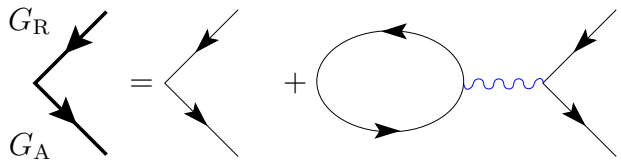


Figure 3: The caption for this figure.

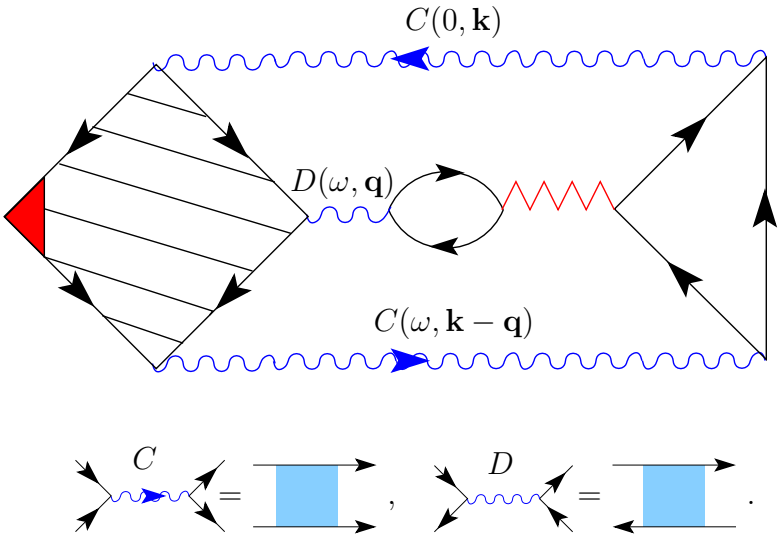


Figure 4: The caption for this figure.